Big Challenges and Big Opportunities: The Power of 'Big Ideas' to Change Curriculum and the Culture of Teacher Planning

Chris Hurst
Curtin University
<c.hurst@curtin.edu.au>

Mathematical knowledge of pre-service teachers is currently 'under the microscope' and the subject of research. This paper proposes a different approach to teacher content knowledge based on the 'big ideas' of mathematics and the connections that exist within and between them. It is suggested that these 'big ideas' should form the basis of teacher planning but it is acknowledged that this represents a 'cultural change'. The proposal is supported by results from a project that involved pre-service teachers in their final mathematics education unit. Results suggest that a focus on the 'big ideas' of mathematics has the potential to change teacher planning and enhance content knowledge.

In recent times, there has been an on-going debate and discussion about teacher and pre-service teacher (PST) competencies and content knowledge. Related issues such as the need for teachers to cover a crowded curriculum while feeling the impact of high stakes testing have added to the discussion. Media releases from Australian Government ministers (Government of Australia, 2013) followed by responses from involved parties such as the Council of Deans of Education indicated that there was broad support for addressing the issues mentioned above. Callingham, Chick and Thornton (2012) had previously noted the growing level of support for some sort of action following the release of results from the Teacher Education and Development Study in Mathematics (TEDS-M) which had highlighted concerns about the level of teacher knowledge for teaching mathematics (Tatto et al., 2008). During 2014, this discussion culminated in the Australian Government's announcement of a Review of the Australian Curriculum along with the establishment of a committee to provide advice about how teacher education programmes could be better structured. (Australian Government, Department of Education, 2014).

Background: A Rationale for Change

Amid the call for better quality teachers, two ideas have been commonly put forward. One is that teacher education degrees should become postgraduate courses following the awarding of a degree in say, mathematics. Another is the use of explicit teaching. Both are laudable ideas and the latter in particular is something that effective teachers may well have been doing anyway. However, it is suggested here that a new approach based on the 'big ideas' of mathematics is needed to enable teachers to deal better with curriculum requirements. It has been noted by Siemon, Bleckley and Neal (2012) that there is a need to 'thin out' the overcrowded curriculum by focussing on the 'big ideas' and promoting a more connected view of mathematics. The situation is similar in the USA where the introduction to the Common Core Standards for Mathematics states that the standards "must address the problem of a curriculum that is a mile wide and an inch deep" (National Governors Association Centre for Best Practices, Council of Chief State School officers, 2010).

Explicit teaching with its modelling and focused questioning should certainly be of benefit as would a greater knowledge of content gained through a dedicated degree;

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* pp. 287–294. Sydney: MERGA.

however, neither is likely to solve the problem on its own. Rather than be concerned with the amount of mathematical knowledge needed by primary teachers, it may be more appropriate to consider how the knowledge is held (Hill & Ball, 2004, cited in Clarke, Clarke, & Sullivan, 2012). It is time to re-conceptualise the mathematical knowledge needed by teachers in terms of the myriad connections and links that exist within and between mathematical ideas. If teachers can be encouraged to understand these connections and links and focus on the 'big ideas' of mathematics there is the potential to revolutionise the way in they think about mathematics and plan for its teaching.

The notion of 'big ideas' is not new and has been most recently discussed by Charles (2005) and Siemon et al. (2012). Charles (2005, p. 10) defines a 'big idea' as "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole". He contends that 'big ideas' are important because they enable us to see mathematics as a "coherent set of ideas" that encourage a deep understanding of mathematics, enhance transfer, promote memory and reduce the amount to be remembered (Charles, 2005). Similarly, the idea of drawing connections has been well documented. Schulman in his seminal paper about knowledge growth discussed "substantive structures [as being the] ways in which the basic concepts and principles of the discipline are organised to incorporate its facts" (Schulman, 1986, p. 9). These 'structures' could be said to be akin to the links and connections of 'big ideas'. Later, Hiebert and Carpenter (1992) noted how understanding depends on a 'network of representations' and Ma (1999) identified 'knowledge packages' where ideas are connected through 'concept knots'. Similarly, Askew, Brown, Rhodes, Wiliam and Johnson (1997) found that the most effective teachers were those who taught from a 'connectionist' standpoint while Barmby, Harries and Higgins (2010) also underlined the importance of 'connections' in developing a deep understanding of mathematical ideas.

A more recent work by Askew (2008) found that there was little evidence to support the notion that very high levels of teacher content knowledge actually benefited children at primary or elementary levels. He is critical of how mathematical content knowledge is reduced to lists of specific pointers that he terms "death by a thousand bullet points" saying that "too much effort goes into specifying the knowledge that teachers need to know" (Askew, 2008, p. 21). The ultimate result is likely to be a continuation of more of the same in terms of curricula. Rather, Askew calls for "a mathematical sensibility ... that would enable them to deal with existing curricula but also be open to change" (2008, p. 22). It is asserted here that his notion of 'sensibility' is akin to having a feel for the 'big ideas' of mathematics and being able to learn about new aspects of mathematics as connections become obvious. Teachers who have such 'sensibility' are likely to be better able to make mathematical connections explicit for their students.

Gojak (2013) noted that the time has come to change the way in which we view elementary/primary mathematics education noting that children need to be taught by teachers who deeply understand mathematics concepts. The inference is that teaching must be done from a conceptual standpoint and perhaps based on 'big ideas' and connections rather than from a traditionally procedural stance. This is supported by Clark's (1997) discussion of concepts that is well encapsulated here:

> My working definition of "concept" is a big idea that helps us makes sense of, or connect, lots of little ideas. Concepts are like cognitive file folders. They provide us with a framework or structure within which we can file an almost limitless amount of information. One of the unique features of these conceptual files is their capacity for cross-referencing (Clark, 1997, p. 94)

Clark cites the work of numerous educators and researchers such as Bruner, Symington and Novak, Brooks and Brooks, and Roszak in describing the power of linkages and the capacities of associations to promote sense making and transfer of learning. Clearly, these ideas have been promoted for some time. It is interesting that Clark equates the term 'concepts' with 'big ideas' and notes how they "provide the cognitive framework that makes it possible for us to construct our own understandings"(Clark, 1997, p. 98). It is suggested here that the focus for developing better teacher knowledge needs to be on the 'big ideas' of mathematics and the links and connections within and between them. This is supported by the research now described.

Research Methodology

The research focused on work done in the final mathematics education unit by a cohort of 64 third and fourth year undergraduate pre-service teachers (PSTs) in the primary/elementary program of one Australian university and sought to understand the potential of the 'big ideas' in mathematics to enhance the content knowledge of those PSTs. This is embodied in the following research question:

- To what extent can a focus on the 'big ideas' of mathematics assist pre-service teachers to develop a deeper understanding of mathematics and the mathematics curriculum as well as their knowledge for planning to teach mathematics?

Data were generated from several sources, namely three aspects of the unit assessment tasks. PSTs were required to develop a 'big idea' concept map and associated rationale, describe links between the 'big idea' and the Australian Curriculum: Mathematics (AC: M), and develop a selection of learning activities chosen because of their link to the 'big idea'. Participants were 'de-identified' and are referred to by pseudonyms. Also, a 6 point Likert Scale questionnaire requiring responses to nine statements about mathematical knowledge and planning for teaching was administered to gain a perspective on how the 'big ideas' focus affected the views of the PSTs about mathematics and in particular about planning for teaching it. A limited discussion of the questionnaire results is included.

Data Analysis

The concept maps were analysed to see the extent to which participants could identify connections within a 'big idea', as well as between it and other ideas. The analysis of the tables of curriculum links focused on the ability of participants to explicitly identify curriculum content descriptors that matched aspects of their rationale and concept map. The rationale statements were analysed manually using key words and phrases to identify emergent themes. Specifically, the analysis focused on the extent to which the rationale statements reflected an understanding of the connections that exist in mathematics and how this can assist in planning to teach mathematics. There were two aspects to the research question that are considered separately, although they are clearly related—knowledge of mathematics and knowledge for planning for teaching mathematics. The concept maps and curriculum links table relate more to the first aspect, whilst the rationale statement relates more to the second aspect.

Results and Discussion

Knowledge of Mathematics (from Concept Maps and Curriculum Link Tables)

A number of general observations can be made following the analysis of the concept maps and curriculum tables. These are listed below and a combined discussion follows:

1. Concept maps depicted two broad types of ideas— ‘content based big ideas’ (n=53) and ‘umbrella big ideas’ (n=11).
2. All PSTs identified a ‘big idea’ and described multiple connections within it.
3. All but five PSTs identified multiple connections to other ‘big ideas’.
4. All PSTs identified a range of activities linked to the AC: M in various content strands that would develop aspects of their ‘big idea’.

First, a wide variety of ‘big ideas’ were considered by the PSTs. The majority could be termed ‘content based’ as they emanate from, or are broadly situated within, one of the content strands of the AC: M. Such ‘big ideas’ were Measurement (n=14), Base Ten Numeration System/Place Value (n=10), Shapes and Solids (n=8), and Chance and Probability (n=6). Others in this group included Fractions and Decimals, Numbers/Number and Algebra, Data, Operations Meanings and Relationships, and Orientation and Location (each n=2) and Space, Mental Mathematics, Transformations, Multiplicative Thinking, and Area (each n=1). Other ideas are termed ‘umbrella big ideas’ as they encompass or are embedded in a number of content areas and such ideas chosen were Pattern (n=7), Comparison (n=2), and Financial Literacy and Equivalence (n=1 each). It is worth noting that while the PSTs had been exposed to the article by Charles (2005) about ‘big ideas’ not one of them selected one of Charles’ ideas and analysis per se but rather approached the ‘big ideas’ from their own standpoint. This reflects the point made by Clarke, Clarke and Sullivan (2012, p. 15) that the value of ‘big ideas’ is found in the way in which they stimulate each teacher (and PST) “to deconstruct her/his own conceptual structures”.

Second, the extent of connections within the ‘big ideas’ identified by the PSTs was great and the examples shown in Table 1 are typical of the number of connections that all PSTs were able to show.

Table 1
Examples of Connections within Selected ‘Big Ideas’

Big Idea and PST	Connections identified within the idea
Base Ten Numeration System (PST Cassie)	Ordering and comparing numbers, flexible partitioning of numbers, additive thinking, patterns in reading and writing numbers, multiplicative thinking, subitising.
Measurement (PST Bronwyn)	Formal and informal language, standard and non-standard units, recording measurement data, benchmarks and referents, measurement principles, appropriate tools and units, real and relevant contexts.
Shapes and Solids (PST Joe)	Relating 2D to 3D, different views of objects, making and using nets, measuring attributes, regular and irregular shapes, position, location and transformation, making models, comparative language.

Third, PSTs identified many ways in which their 'big ideas' were connected to other 'big ideas' and also indicated how these links were present in the Australian Curriculum through various activities. Figure 1 represents one 'big idea' (Measurement) and shows how various PSTs drew connecting pathways between it and other ideas. The direction of arrows is indicative of the 'big idea' that was the 'source idea'.

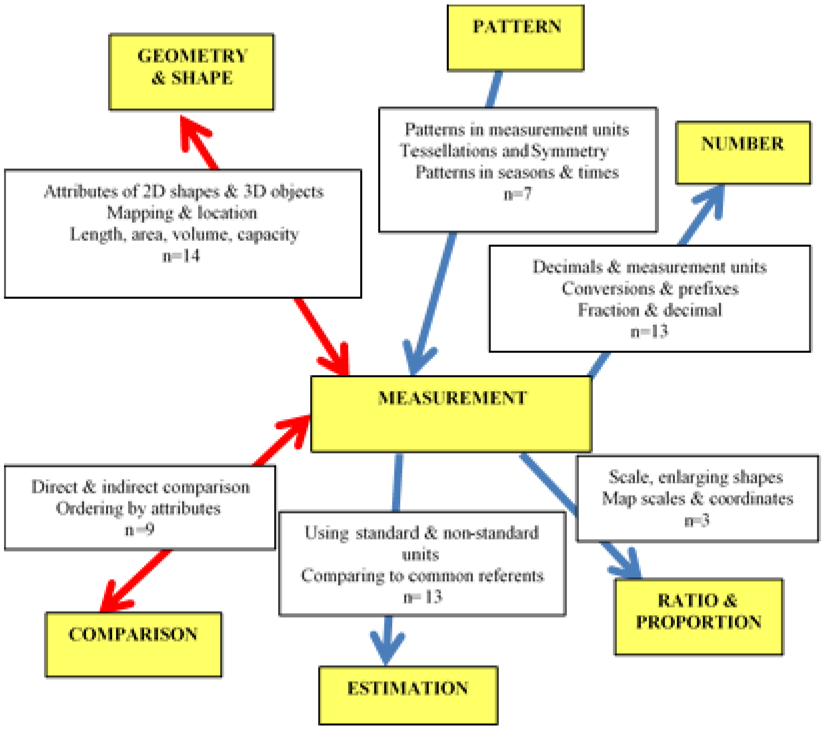

Figure 1. Summary of main pathways connecting various 'big ideas'.

The different pathways drawn by different PSTs support the earlier comment attributed to Clarke et al. (2012). It underlines how the process of constructing one's content knowledge is likely to differ greatly from person to person (Clarke et al., 2012). It is also encapsulated well in Table 2 which shows how three PSTs linked their 'big idea' of Pattern to other 'big ideas' through a task shown in their curriculum link tables.

Table 2
Links between the 'big idea' of Pattern and the Australian Curriculum: Mathematics

PST	Links between Pattern and the Australian Curriculum: Mathematics
Penny	Explore and colour patterns in skip counting and multiples using a 1-200 number grid and a basic facts grid. Generate patterns with the constant function of a calculator (Number and Algebra).
Sally	Investigate patterns in reading timetable and reading/writing clock times. Investigate patterns in shadow length (Measurement). Describe toothpick patterns and make generalisations (Number and Algebra).
Dan	Explore patterns in symmetry of 2D shapes (Geometry). Collect, organise and represent data and explore patterns in graphs. Explore patterns in music, time and weather and make predictions based on patterns (Statistics and Probability).

Knowledge of Planning for Teaching Mathematics (from Rationale Statements and Questionnaire Responses)

A number of strong themes emerged from the analysis of the PSTs' rationale statements in which PSTs suggest that a focus on 'big ideas' assists in the following ways:

1. Promotes greater understanding of mathematical ideas and sense-making
2. Clarifies mathematical connections and links
3. Promotes transfer of ideas
4. Clarifies ways of dealing with a compartmentalised and crowded curriculum
5. Promotes planning across year levels by clarifying how ideas develop
6. Enables them to more effectively teach from a conceptual standpoint

The following annotated examples of comments made in the rationale statements are typical in that they relate to more than one of the above themes as indicated.

> Mathematics shouldn't be a study of disconnected facts and chunks of meaningless information to memorise but rather a place [where] students learn to connect what they already know in an ever-increasing network of maths ideas and skills [Theme 2]. Teachers who understand the big ideas of mathematics can more easily demonstrate links between different areas of mathematics, and help students to make connections as they learn. This interconnected view of mathematics emphasises the development of conceptual understanding [Theme 6] ... that focuses less on procedures and more on sense-making [Theme 1]. (PST Eva)

In her rationale, PST Eva also made repeated mention of how the 'big ideas' focus would encourage children to make sense of their mathematical learning, to be able to approach problems from a range of viewpoints and see multiple solutions.

> The mathematics curriculum has always been nicely categorised into units and specific content areas which are then taught independently to students. This teaching of the content areas and strands independently does not provide students with the opportunity to understand mathematics as a whole and see the interconnected ideas and processes that underpin mathematical knowledge and thought [Theme 4]. By teaching mathematics in a more streamlined manner teachers are able to show students the main ideas and the links between them [Theme 2]. (PST Brianne)

PST Brianne's discussion of curriculum is supported by PST Toni's comments. She noted that to learn vital measurement concepts, children must learn ideas about conversion, comparison, connection and calculation of numbers and units, which are covered in other content descriptors, many of which "have a tendency to overlap" [Theme 4] (PST Toni).

> Big ideas is a very effective method of fostering a deep understanding of mathematics [Theme 1]. Not only does it facilitate learning for a wide diversity of students but it authentically links seemingly isolated facts [Theme 2] to provide meaning and transfer and develops skills that allow students to effectively apply their learning to new situations [Theme 3]. (PST Bella)

PST Bella also stated that a 'big ideas' focus fosters conceptual learning and trans-disciplinary connections which effectively facilitates understanding [Theme 6].

> Knowing how one big idea links to another helps the teacher and the student understand why certain concepts need to be learnt or taught [Theme 6]. Teachers who know about big ideas also know that they link across year levels and know how the concepts and skills develop at each year level as well as how they connect to previous and following year levels [Theme 5]. (PST Sue)

It is significant that PST Sue and PST Jenny (following) made the connection between enhanced teacher knowledge and improved student understanding with Sue emphasising the 'purposeful' aspect of learning about certain concepts and Jenny noting the vital importance of key underpinning 'big ideas' in promoting conceptual understanding.

> Comparison is an early concept that is a stepping stone to more complex and elaborate mathematical ideas. Every time a new concept is introduced within the Australian Curriculum: Mathematics, comparison is used to explore and familiarise students with that particular concept [Theme 5]. Comparison is a foundational concept, and without correct development, could greatly impede a student's development big ideas and concepts in years to come [Theme 2]. (PST Jenny)

PST Shay's comments echoed those of other PSTs about connections, sense-making and purpose, noting that 'big ideas' help children "to see new ways of expressing mathematical ideas, making connections and sense of various mathematical ideas [Theme 2] and most importantly creating the crucial link between all areas [Theme 3] in mathematics which delivers a sense of meaning and purpose to students' learning" [Theme 1]. (PST Shay)

The questionnaire responses illuminated similar themes to those from the rationale statements but provided greater insight into how the 'big ideas' focus had influenced PSTs' thinking about teaching children, their understanding of curriculum and content, and their personal readiness for teaching. Within the theme about 'Teaching children better', two aspects emerged, the first being related to 'teaching for understanding'. The most common responses were that the 'big ideas' focus "Helped me to understand how to teach mathematics for understanding" (n=29) and "Helped me clarify understanding for teaching mathematics" (n=21). The second aspect related to 'how children learn' and the most common responses were "Realised how ideas are built from prior knowledge of other related ideas" (n=16), "Clarity of learning trajectories has given [me] more confidence" (n=7) and "Better able to know about student learning and misconceptions" (n=5).

With regard to understanding curriculum content, the responses clearly focused on the notion of connections. The most common responses were "Connectedness of ideas makes it easier to organise and teach content and better able to help children connect ideas" (n=22), "Seeing the interconnectedness of ideas gives more confidence to teach" (n=11) and "Interconnectedness makes more sense now" (n=10). Regarding personal readiness for teaching, responses reiterated notions of confidence derived from better understanding. The main responses were "Have more understanding and are better prepared to plan and teach mathematics" (n=33), "Greater conceptual understanding gives more confidence to teach" (n=16) and "Have greater sense of clarity and insight about mathematics" (n=7).

Conclusion

The results presented provide clear evidence that, given the opportunity, PSTs are certainly capable of thinking about mathematics in a conceptual way based on the connections and links within and between 'big ideas' of mathematics. The extent of the links identified by the PSTs was considerable, particularly between different 'big ideas' and not only did the PSTs represent these links and connections but they also reported that they realised how they could be of great benefit in their planning for teaching. Moreover, they acknowledged how a focus on 'big ideas' enabled them to consider the Australian Curriculum: Mathematics in a different way. They no longer felt constrained by the linear structure of the content but could see how mathematical ideas are best developed across a number of year levels. Perhaps most importantly, they could see that planning to teach mathematics in this way would have multiple benefits for their students.

The 'big ideas' focus certainly presented mathematics in a different way to what the PSTs had been accustomed. A number of them commented to the effect that it had challenged their thinking and that it was initially quite daunting to consider mathematics in such a way. However, as the unit progressed, they began to feel genuinely excited by the prospect of thinking about mathematics and teaching it in this way. Most importantly, they noted how the 'big ideas' focus enabled them to help children better through using links and connections to different concepts to overcome misconceptions and misunderstandings. The following questionnaire response encapsulates much of the learning that took place.

> Prior to learning about the big ideas I was aware that the maths curriculum is content heavy & therefore worried about how to teach children mathematical understanding & reasoning effectively. I now see that relating it to big ideas makes maths more linked & provides opportunities for greater conceptual coverage of the curriculum.

The success of 'big ideas' focus with this cohort of PSTs has strong implications for how we train teachers and provide professional learning for in-service teachers.

References

Askew, M. (2008). Mathematical discipline knowledge requirements for prospective primary teachers, and the structure and teaching approaches of programs designed to develop that knowledge. In P. Sullivan & T. Woods (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development, Volume 1* (pp. 13-35). Rotterdam: Sense Publishers.

Askew, M., Brown, M., Rhodes, V., Wiliam, D., & Johnson, D. (1997, September). *Effective teachers of numeracy in primary schools: Teachers' beliefs, practices and pupils' learning*. Paper presented at the British Educational Research Association Annual Conference, September 1997: University of York.

Australian Government: Department of Education. (2014). *Review of the Australian Curriculum.* Retrieved from: http://www.studentsfirst.gov.au/strengthening-australian-curriculum

Barmby, P., Harries, T., & Higgins, S. (2010). Teaching for understanding / understanding for teaching. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools* (2nd ed.) (pp. 45-57). Berkshire, UK: Open University Press.

Callingham, R., Chick, H., & Thornton, S. (2012). Editorial comment. *Mathematics Teacher Education and Development, 14*(2), 2-3.

Charles, R. I. (2005). Big ideas and understandings as the foundation for early and middle school mathematics. *NCSM Journal of Educational Leadership, 8*(1), 9–24.

Clark, E. (1997). *Designing and implementing an integrated curriculum: A student-centred approach.* Brandon, Vermont: Holistic Education Press.

Clarke, D. M., Clarke, D. J., & Sullivan, P. (2012). Important ideas in mathematics: What are they and where do you get them? *Australian Primary Mathematics Classroom, 17*(3), 13-18.

Gojak, L.M. (2013). *It's elementary! Rethinking the role of the elementary classroom teacher.* Retrieved from: http://www.nctm.org/about/content.aspx?id=37329

Government of Australia. (2013, March 11). *Higher standards for teacher training courses* [Press Release]. Retrieved from: http://ministers.deewr.gov.au/garrett/higher-standards-teacher-training-courses

Hiebert, J. & Carpenter, T.P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: MacMillan.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.

National Governors Association Center for Best Practices, Council of Chief State School Officers (NGA Center). (2010). *Common core state standards for mathematics.* Retrieved from: http://www.corestandards.org/the-standards

Schulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4-14.

Siemon, D., Bleckly, J., & Neal, D. (2012). Working with the big ideas in Number and the Australian Curriculum: Mathematics. In B. Atweh, M. Goos, R. Jorgensen, & D. Siemon, (Eds.). *Engaging the Australian National Curriculum: Mathematics – Perspectives from the Field* (pp. 19-45). Online Publication: Mathematics Education Research Group of Australasia.

Tatto, M. T., Schwille, J., Senk, S., Ingvarson, L., Peck, R., & Rowley, G. (2008). *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, practice, and readiness to teach primary and secondary mathematics. Conceptual framework.* East Lansing, MI: Teacher Education and Development International Study Center, College of Education, Michigan State University.